Computer-Aided Synthesis of Planar Circuits

FUMIO KATO, MASAO SAITO, MEMBER, IEEE, AND TAKANORI OKOSHI, MEMBER, IEEE

Abstract—**This paper presents a fully computer-oriented iterative synthesis of an open-boundary planar circuit having an impedance matrix with prescribed poles and residues. A starting circuit pattern is given first, and it is represented by a finite number of parameters. Those parameters (and hence, the pattern) are then iteratively modified by using the Newton–Raphson method to realize the prescribed circuit characteristics. When the numbers of given poles and coupling ports are relatively small, the results are satisfactory both in the computing time and accuracy. Some numerical examples are given.**

I. Introduction

THE PLANAR (two-dimensional) circuit is a circuit category that should be positioned between the distributed-constant circuit and the waveguide circuit [1]. As this concept was originally proposed for rigorous analysis and design of microwave and millimeter-wave IC's, its synthesis (determination of the circuit pattern giving prescribed circuit characteristics) is an important technical task.

However, although a number of methods have been presented for the analysis [1]–[4], rather few papers have ever dealt with the synthesis. Okoshi *et al.* described a trial-and-error synthesis of a ladder-type 3-dB hybrid consisting of wide striplines [5]; their synthesis, however, is never of a general nature. Grüner dealt with the conformal-mapping synthesis of a thin waveguide section [6]. Since a thin waveguide section can be regarded as a short-boundary planar circuit, his method will also be applicable, if appropriately modified, to open-boundary planar circuit. However, Grüner described synthesis of only poles of transmission characteristics. Synthesis of residues must also be performed to make the synthesis complete.

This paper presents a fully computer-oriented iterative synthesis of an open-boundary planar circuit having an impedance matrix with prescribed poles and residues. Basically, this paper's aims are similar to Grüner's [6]; the differences are: 1) open-boundary problems are considered, 2) prescribed residues are realized in addition to poles, and 3) the mapping technique has been improved.

II. Basic Equations and Principle of Analysis

A. Circuit Equations and Impedance Matrix

Consider a planar circuit as shown in Fig. 1. It consists of a conductive (circuit) plate with an arbitrary shape, an isotropic dielectric layer, and a ground plate.[1] The boundary of the circuit pattern is denoted by Γ, and its interior is henceforth called the circuit pattern and is denoted by S. The spacing d between the circuit plate and the ground plate is assumed to be much less than the wavelength. The circuit is assumed to be lossless.

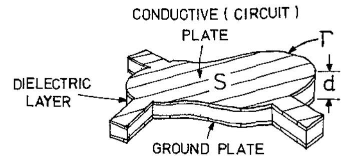

Fig. 1. An open-boundary planar circuit.

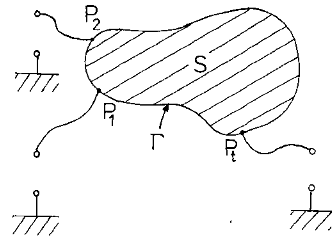

Fig. 2. Schematic representation of a multiport planar circuit.

The steady-state circuit equation is then given by the following two-dimensional Helmholtz equation:

$$\nabla^2 v(x,y) + \lambda v(x,y) = 0 \tag{1}$$

where $\lambda = \omega^2 \varepsilon\mu$, ε and μ are the permittivity and permeability of the spacing material, respectively, and ∇^2 denotes the Laplacian [1]. Upon Γ where no coupling port is present, the boundary condition is given as

$$\frac{\partial v}{\partial n} = 0 \tag{2}$$

where $\partial/\partial n$ denotes the derivative normal (outward) to Γ. Equations (1) and (2) give a countably infinite number of eigenvalues λ_i and associated normalized eigenfunctions $v_i(x,y)$.

When t ports are connected to the circuit plate, as shown in Fig. 2, at positions $P_1, P_2, \cdots, P_t$ and assumed to have negligible widths as compared with the wavelength, the (m,n)

Manuscript received April 28, 1976; revised March 1, 1977.

F. Kato and T. Okoshi are with the Department of Electronic Engineering, Faculty of Engineering, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113, Japan.

M. Saito is with the Department of Medical Electronics, Faculty of Medicine, University of Tokyo, 7-3-1 Hongo, Bunkyo-ku, Tokyo 113, Japan.

[1] In the earlier papers [1]–[4], triplate-type planar circuits have been mainly dealt with. In this paper a two-plate circuit as shown in Fig. 1 is considered for its simplicity. Mathematically the above two types are equivalent, except that in the former the impedance level is reduced by one half because currents flow in both the upper and lower surfaces of the circuit plate.

element of the impedance matrix of the t-port circuit is expressed in terms of λ_i and v_i as [1]

$$Z_{mn} = \sum_{i=0}^{\infty} \frac{-j\omega\mu d}{\lambda - \lambda_i} v_i(P_m) v_i(P_n). \quad (3)$$

We should note that λ_i's correspond to poles of Z_{mn} and that $v_i(P_m)$ and $v_i(P_n)$ determine the residue matrix associated with λ_i.

B. Approximate Solution by Rayleigh–Ritz Method

An approximate solution (λ_i and v_i) of (1) can be obtained from the stationary condition of the functional

$$I(v,\lambda) = \iint_S \left\{ \left(\frac{\partial v}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial y}\right)^2 - \lambda v^2 \right\} dS. \quad (4)$$

Let $v(x,y)$ be expanded approximately by a truncated series

$$v(x,y) = \sum_{k=1}^{M} a_k f_k(x,y) \quad (5)$$

where $\{f_k(x,y)\}$ is a system of functions which is complete in the region S.

We rewrite the variational problem, (4), into a form which is more suitable to the computer analysis. By using

$$A_{kl} = \iint_S \nabla f_k \cdot \nabla f_l \, dS \quad (6a)$$

$$B_{kl} = \iint_S f_k f_l \, dS, \qquad k, l = 1, 2, \cdots, M \quad (6b)$$

$$A = \begin{bmatrix} A_{11} & \cdots & A_{1M} \\ \vdots & & \vdots \\ A_{M1} & \cdots & A_{MM} \end{bmatrix} \qquad B = \begin{bmatrix} B_{11} & \cdots & B_{1M} \\ \vdots & & \vdots \\ B_{M1} & \cdots & B_{MM} \end{bmatrix} \quad (7)$$

(4) is reduced to a matrix eigenvalue problem

$$(A - \lambda B)a = 0 \quad (8)$$

where a denotes a column vector consisting of the coefficients $a_1, a_2, \cdots, a_M$ in (5). Thus, from the nontrivial condition of (8), the eigenvalues can be obtained as the roots of the characteristic equation

$$\det(A - \lambda B) = 0. \quad (9)$$

The corresponding eigenvector a is then obtained from (8) except for a constant multiplier, which is determined by using the normalizing condition

$$\iint_S v^2(x,y) \, dS = 1. \quad (10)$$

In the actual computation, the constant multiplier of a is determined by the matrix equivalence of (10):

$$a^T B a = 1 \quad (11)$$

where superscript T denotes transposition. Finally, v is given in terms of a by using (5).

III. Method of Synthesis

In the following synthesis, we must represent the circuit pattern and port locations with a finite set of parameters. However, whatever parameters are employed, the essential process of the synthesis remains unchanged. Hence, we begin this section with a general description of the synthesis without specifying any particular type of parameters. In Section III-D, two practical choices of the parameters will be presented.

A. Modified Newton–Raphson Method

We first describe the iterative process used in the synthesis. Consider a system of n equations to be solved:

$$F(X) = \begin{bmatrix} F_1(X) \\ \vdots \\ F_n(X) \end{bmatrix} = 0 \quad (12)$$

where X denotes a column vector whose elements are $N (N \geq n)$ unknown quantities representing the circuit pattern, that is, $X = (X_1, X_2, \cdots, X_N)^T$. The Jacobian matrix of $F(X)$ with respect to X is expressed as

$$J(X) = \begin{bmatrix} J_{11}(X) & \cdots & J_{1N}(X) \\ \vdots & & \vdots \\ J_{n1}(X) & \cdots & J_{nN}(X) \end{bmatrix} \quad (13)$$

where

$$J_{ij}(X) = \frac{\partial F_i(X)}{\partial X_j}, \qquad i = 1, \cdots, n; \quad j = 1, \cdots, N. \quad (14)$$

We first assume a vector $X^{(0)}$ as the initial pattern, and modify it to obtain the solution of (12). For this purpose, the $(h+1)$th solution should be obtained from the hth solution as [7]

$$X^{(h+1)} = X^{(h)} - J^{-}(X^{(h)})F(X^{(h)}) \quad (15)$$

where J^- represents a generalized inverse matrix of J. Then $X^{(h)}$ should finally converge to X^* which satisfies

$$J^T(X^*)F(X^*) = 0. \quad (16)$$

In particular, when rank $\{J(X^*)\} = n$ as is usually the case,

$$F(X^*) = 0 \quad (17)$$

holds, which is the condition to be satisfied.[2] If we add, in computing $X^{(h)}$, an additional condition (the minimum norm condition)

$$|X^{(h+1)} - X^{(h)}| = \text{minimum} \quad (18)$$

then the modification algorithm, (15), is rewritten as [7]

$$X^{(h+1)} = X^{(h)} - J^T(JJ^T)^{-1}F(X^{(h)}). \quad (19)$$

[2] In order that X converges to the solution, $X^{(0)}$ must satisfy some conditions, which have been discussed in detail by Ben-Israel [7]. However, the convergence is guaranteed if (15) is modified so that the convergence is decelerated [8]. Such a modified equation is used in the computation described in Section IV.

B. Synthesis of Prescribed Poles

For some practical purposes such as the design of a bandpass filter or the suppression of spurious modes in a resonator, synthesis of the poles (eigenvalues) disregarding the residues is useful to some extent [6]. In this subsection, to begin with, eigenfunctions are not considered; a circuit pattern having several prescribed eigenvalues will be synthesized.

We assume that a set of the smallest n eigenvalues $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_n)^T$ $(0 = \lambda_0 < \lambda_1 < \lambda_2 < \cdots < \lambda_n;\ n \leq N)$ are given. From (9), those must satisfy

$$F_i(X) = \det\{A(X) - \lambda_i B(X)\} = 0, \qquad i = 1, \cdots, n. \quad (20)$$

As discussed in the preceding subsection, X is successively modified by the Newton–Raphson method to arrive finally at the solution satisfying $F(X^*) = 0$. We assume that $F_i(X) \neq 0$ and write

$$C_{kl}^i(X) = A_{kl}(X) - \lambda_i B_{kl}(X) \quad (21a)$$

$$C_i(X) = A(X) - \lambda_i B(X). \quad (21b)$$

Matrix $C_i(X)$ is then regular, and J_{ij} in (14) may be rewritten as

$$J_{ij}(X) = F_i(X) \sum_{k=1}^{M} \sum_{l=1}^{M} \frac{\partial C_{kl}^i}{\partial X_j} [C_i]_{kl}^{-1} \quad (22)$$

where $[C_i]_{kl}^{-1}$ represents the (k,l) element of the inverse matrix of C_i. In actual synthesis of a circuit, each modification should be as small as possible to avoid an unreasonable circuit pattern and oscillation of the solution. Therefore we use (19) rather than (15).

The initial circuit pattern $X^{(0)}$ must be chosen with some care. We may reconcile at least one eigenvalue to the desired value merely by multiplying $X^{(0)}$ by a positive constant. Empirically, the convergence seems to be improved by choosing the initial pattern $X^{(0)}$ so that its nth (highest) eigenvalue is equal to the prescribed value.

C. Synthesis of Prescribed Impedance Matrix

We assume t coupling ports along the boundary. The residues of the impedance matrix ($\mu\, dv_i(P_m)v_i(P_n)$ in (3)) are now considered as well as the poles. Let the positions of the ports be denoted by $P_1, \cdots, P_t$; their parameter representations, by $\delta_1, \cdots, \delta_t$. Since the values of eigenfunctions at those ports must be adjusted as well as the eigenvalues, each step of the synthesis must include shifts of the port locations $P_1, \cdots, P_t$. Therefore, the parameters representing a circuit pattern are now

$$X = (X_1, \cdots, X_N, \delta_1, \cdots, \delta_t)^T. \quad (23)$$

The input data now comprise the number of ports t, the prescribed eigenvalues λ_i $(i = 1, \cdots, n)$, and the values of associated eigenfunctions at the kth port q_{ik} $(i = 1, \cdots, n;\ k = 1, \cdots, t)$. Let $\lambda_i(X)$ denote the ith eigenvalue of the circuit pattern X; $v_{ik}(X)$, the value of the corresponding eigenfunction at the kth port. Then we may apply the Newton–Raphson method to the following equations:

$$F(X) = \begin{bmatrix} F_1(X) \\ \vdots \\ F_n(X) \\ F_{n+1}(X) \\ \vdots \\ F_{n(t+1)}(X) \end{bmatrix} = \begin{bmatrix} \lambda_1(X) - \lambda_1 \\ \vdots \\ \lambda_n(X) - \lambda_n \\ v_{11}(X) - q_{11} \\ \vdots \\ v_{nt}(X) - q_{nt} \end{bmatrix} = 0. \quad (24)$$

The Jacobian matrix of $F(X)$ is then obtained from (5), (8), (11), and (24).

D. Parameters Representing Circuit Pattern

In Sections III-B and -C, the synthesis process has been formulated without referring to the implication of the parameters representing the circuit pattern. In this subsection, two choices of the parameters are described.

1) Sector Method: We assume an origin O in the region S and N sampling points along the boundary Γ. The sampling points are chosen so that the angle at the origin subtended by any adjacent two sampling points is equal to $2\pi/N$ (see Fig. 3). We denote the distance between the origin and each sampling point by $X_1, X_2, \cdots, X_N$; the circuit pattern is then represented approximately by $X = (X_1, X_2, \cdots, X_N)^T$. When we finally obtain X_j's satisfying

$$X_j > 0, \qquad j = 1, \cdots, N \quad (25)$$

we may consider X to be physically realizable.

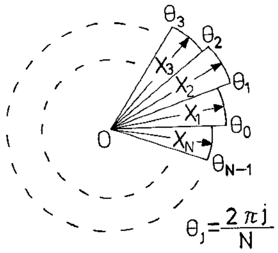

Fig. 3. Representation of the circuit pattern in the sector method.

In the polar coordinates, A_{kl} is given, from (6a), as

$$A_{kl} = \sum_{j=1}^{N} \int_0^{X_j} \int_{\theta_{j-1}}^{\theta_j} \nabla f_k \cdot \nabla f_l r\, dr\, d\theta, \qquad k, l = 1, \cdots, M \quad (26)$$

and its derivative with respect to X_j as

$$\frac{\partial A_{kl}}{\partial X_j} = \int_{\theta_{j-1}}^{\theta_j} \nabla f_k \cdot \nabla f_l r \bigg|_{r = X_j} d\theta. \quad (27)$$

Expressions for B_{kl} and its derivative may be given in similar forms.

We may use the above parameters in either of the syntheses described in Sections III-B and -C. For convenience those combinations will hereafter be called synthesis processes I and II, respectively. In process II, however, we find some difficulty in considering the external ports because of an inherent discontinuous nature of the boundary consisting of many sectors (see Fig. 3).

2) *Conformal-Mapping Method:* Any simply connected region S in the complex z plane can be mapped from the interior of a unit circle Ω in another complex ζ plane (see Fig. 4) by a regular function

$$z = x + jy = \phi(\zeta) \tag{28}$$

which we express by a truncated Maclaurin series as[3]

$$\phi(\zeta) = \sum_{k=1}^{m} (\alpha_k + j\beta_k)\zeta^k, \qquad \alpha_k \text{ and } \beta_k \text{ are real.} \tag{29}$$

Thus the pattern S can be represented by α_k and β_k ($k = 1, \cdots, m$). Further, we define a vector X as

$$X = (X_1, X_2, \cdots, X_{N-1}, X_N)^T = (\alpha_1, \beta_1, \cdots, \alpha_m, \beta_m)^T \tag{30}$$

where

$$N = 2m \tag{31a}$$

$$X_{2k-1} = \alpha_k \qquad X_{2k} = \beta_k, \qquad k = 1, \cdots, m. \tag{31b,c}$$

Equation (1) expressed in the z plane is transformed, in the ζ plane, to

$$\nabla_\zeta^2 v + \lambda |\phi'(\zeta)|^2 v = 0 \left(\nabla_\zeta^2 = \frac{\partial^2}{\partial \xi^2} + \frac{\partial^2}{\partial \eta^2}\right). \tag{32}$$

The boundary condition, (2), remains unchanged because a mapping by a regular function is conformal. In accordance with the above equation, we define A_{kl} and B_{kl} as

$$A_{kl} = \iint_\Omega \nabla f_k \cdot \nabla f_l \, d\xi \, d\eta \tag{33}$$

$$B_{kl} = \iint_\Omega f_k f_l |\phi'(\zeta)|^2 \, d\xi \, d\eta \tag{34}$$

with which the matrix eigenvalue problem is expressed in the same form as (8). Quantities A_{kl} and B_{kl} and their derivatives with respect to X_j can be computed easily because the region of integrals in (33) and (34) is a unit circle. Furthermore, A_{kl}'s are independent of $\phi(\zeta)$; hence

$$\frac{\partial A_{kl}}{\partial X_j} = 0, \qquad j = 1, \cdots, N; \quad k, l = 1, \cdots, M \tag{35}$$

holds.

Let those points in the ζ plane corresponding to the positions of ports $P_1, \cdots, P_t$ be denoted by $Q_1, \cdots, Q_t$, respectively. Those points are all located on the unit circle; hence, their arguments $\delta_1, \cdots, \delta_t$ can be used as the parameters representing the port locations (see Fig. 4).

[3] Instead of (29), Grüner [6] used an expansion of the form $|\phi'(\zeta)|^2 = \exp\{\sum_{k=1}^{N} c_k f_k(\xi,\eta)\}$ where $f_k(\xi,\eta)$ is a harmonic function. Therefore, a differential equation must be solved to determine the mapping function $\phi(\zeta)$ from $|\phi'(\zeta)|^2$. Expansion (29) seems better than Grüner's because mapping function $\phi(\zeta)$ is directly obtained from calculated coefficients as a regular function.

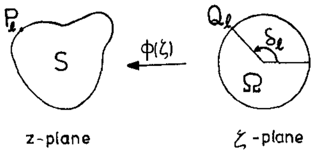

Fig. 4. Conformal mapping from the ζ plane into the z plane.

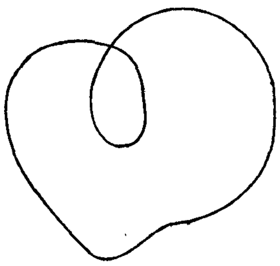

Fig. 5. An example of the circuit pattern without physical realizability.

TABLE I
FUNDAMENTAL FUNCTIONS USED IN THE SYNTHESIS

1	r	r^2	....	r^{n_r}
	$r \sin \theta$	$r^2 \sin \theta$	....	$r^{n_r} \sin \theta$
	$r \cos \theta$	$r^2 \cos \theta$	....	$r^{n_r} \cos \theta$
	⋮	⋮		⋮
	$r \sin n_\theta \theta$	$r^2 \sin n_\theta \theta$	....	$r^{n_r} \sin n_\theta \theta$
	$r \cos n_\theta \theta$	$r^2 \cos n_\theta \theta$	....	$r^{n_r} \cos n_\theta \theta$

We may apply the above conformal-mapping method to either of the syntheses described in Sections III-B and -C. They will be called synthesis processes III and IV, respectively. A problem in these processes is that the multivalent region, shown in Fig. 5, sometimes appears as the result of mapping. To prevent this, at least

$$\phi'(\zeta) \neq 0 \tag{36}$$

must hold in Ω. This condition is not always satisfied easily.

IV. EXAMPLES OF SYNTHESIS

Examples of synthesis based upon processes I–IV will be described. The fundamental functions used are shown in Table I, where $n_r = 4$ and $n_\theta = 2$.

A. Synthesis Processes I and II

The given parameters are: $N = 32$, $n = 3$, $\lambda_1 = 3.4$, $\lambda_2 = 7.0$, and $\lambda_3 = 7.4$. The successive modification of the circuit is shown in Fig. 6 for process I and in Fig. 7 for process II. Fig. 8(a) and (b) depicts how the lowest three eigenvalues of the circuit approach the prescribed values as the analysis and modification are iterated for process I and II, respectively. In Fig. 8 the horizontal solid lines indicate the prescribed eigenvalues.

These examples show that process I produces a smoother pattern than process II, but much more iteration is needed to

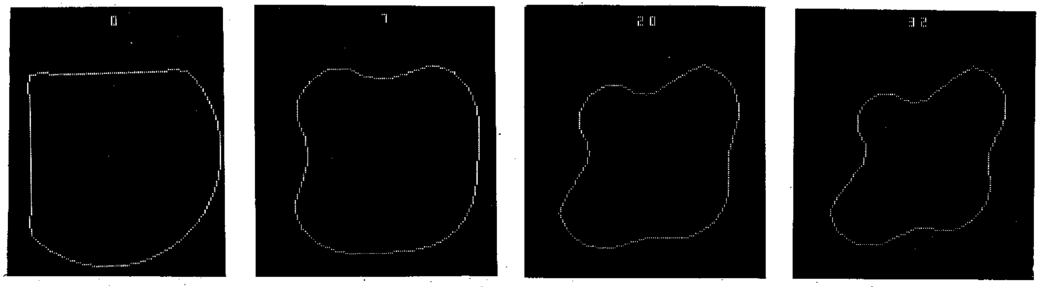

Fig. 6. Circuit patterns for iteration numbers $h = 0$, 7, 20, and 32 by process I (graphic output of the computer).

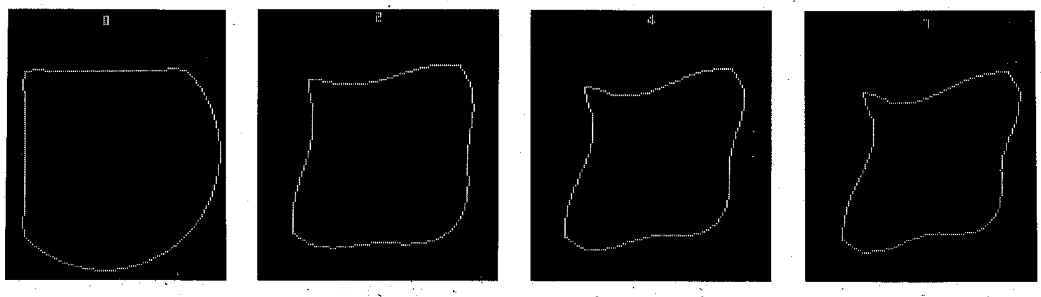

Fig. 7. Circuit patterns for iteration numbers $h = 0$, 2, 4, and 7 by process II (graphic output of the computer).

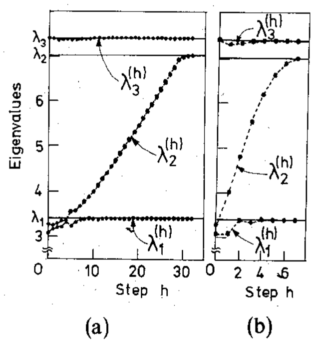

Fig. 8. The behavior of eigenvalues. (a) The case of Fig. 6. (b) The case of Fig. 7.

reach the prescribed eigenvalues. The above statement is valid for almost all cases. The computer time required for one cycle of analysis and modification was about 2 s in both processes when HITAC-8800 was used.

B. Synthesis Processes III and IV

Synthesis processes III and IV were applied to a similar problem, in which $n = 3$, $\lambda_1 = 3.5$, $\lambda_2 = 5.0$, and $\lambda_3 = 7.0$. The degree of the mapping function $m = 16$, that is, $N = 32$. The details of the modification processes are omitted for the sake of brevity.

The computer time for one cycle was about 2.5 s in both processes. The differences in the final pattern and in the speed of convergence were not remarkable for the two processes. However, processes III and IV are inferior to processes I and II in that the behavior of the convergence is somewhat oscillatory. Besides, in some cases, a univalent mapping could not be found. These methods, therefore, should better be employed only when external coupling ports must be considered.

Therefore, in this subsection, synthesis of an impedance matrix of a two-port planar circuit using process IV will be described in more detail. Two eigenvalues $\lambda_1 = 4.0$ and $\lambda_2 = 5.0$ are specified, and the prescribed values of the associated eigenfunctions at the ports are given as, $q_{11} = 0.95$, $q_{12} = 0.95$, $q_{21} = -0.7$, and $q_{22} = 1.6$ (see (24)). Figs. 9 and 10 show the modification process and the convergence of parameters, respectively. In Fig. 9, P_1 and P_2 denote the port locations. The computer time required for one step was a little less than 2 s.

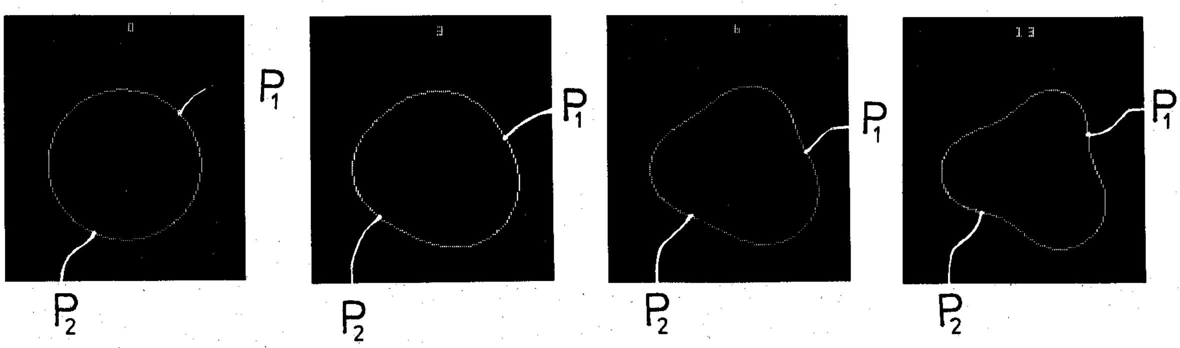

Fig. 9. Circuit patterns and port locations for iteration numbers $h = 0, 3$, 6, and 13 where P_1 and P_2 indicate port 1 and port 2, respectively (graphic output of the computer).

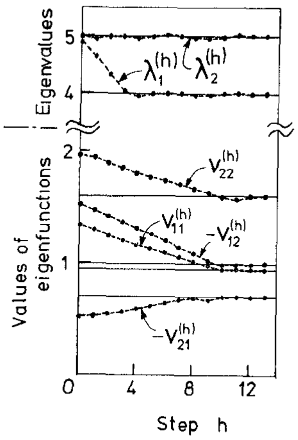

Fig. 10. The behavior of eigenvalues and the values of eigenfunctions at the ports in the case of Fig. 9.

V. CONCLUSION

A basic algorithm and numerical examples of the synthesis of planar circuits have been presented. When the number of the prescribed eigenvalues and external ports is relatively small, the results are satisfactory both in the computing time and accuracy. However, research is still at a primitive stage; further efforts are needed to make the proposed method practical, especially for larger numbers of eigenvalues and ports. In synthesis processes III and IV, the problem of the multivalent region must be overcome to make those methods practical.

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Equivalent Circuit Capacitance of Microstrip Step Change in Width

CHANDRA GUPTA, STUDENT MEMBER, IEEE, AND ANAND GOPINATH, MEMBER, IEEE

***Abstract*—Calculated results which extend existing data on the capacitance of step discontinuity are presented for w_1/H of value 0.1, 0.5, 1.0, and 2.0, for relative dielectric constants of 15.1, 9.0, 4.0, and 2.3, and for w_2/H in the range 0.1–10.0. The quasi-static method of calculation is used, and the excess capacitance associated with the steps is determined by the solution of the integral equation using Green's functions.**

INTRODUCTION

THE RANGE of data currently available on the microstrip discontinuities is inadequate, thus microstrip circuit designs are currently implemented after a few trial stages. The present paper extends the range of the capacitances of the step change in width discontinuity beyond that provided by Farrar and Adams [1] and Benedek and Silvester [2]. The calculations performed for this data utilize the integral equation approach using Green's functions and the concept of "excess charge" due to Benedek and Silvester [2] to preserve numerical accuracy. The method of solution discretizes the discontinuity into rectangular elements and the excess charge is obtained by the Galerkin method.

Radiation and dispersion effects are neglected, and, therefore, the microstrip discontinuity problem may be reduced to a quasi-static form. The stored energy of the step discontinuity may then be represented by an equivalent circuit in the form of a T circuit, given in Fig. 1(b), for the chosen reference planes TT'. The present calculations evaluate the shunt capacitance of this circuit, the inductive components have been presented elsewhere [3].

In the following sections, we briefly outline the formulation and the method of solution followed by the results.

Manuscript received December 10, 1976; revised March 1, 1977. This work was supported by the U.K. Ministry of Defence, Procurement Executive under the CVD Directorate and is published with permission. C. Gupta is supported by a UCNW scholarship.

The authors are with the School of Engineering Science, University College of North Wales, Bangor, Gwynedd LL57 1UT, U.K.